10.6 CMB Method

10.6.1 Overview

The supposition of simple mass conservation is adopted in the basic concept of the receptor model, which is used to determine the emission source of the suspended particulate matter in the atmosphere. That is, the emission source exists in the symbol p and supposes that there is not an interaction which would cause the removal and generation of a mass among the discharged particulate matter. Then, the material concentration $C$ of the particle in the receptor (the measuring point) becomes the linear summation of $S_j$ from each emission source as seen in Eq. (22).

$$C = \sum_{j=1}^{p} S_j \tag{22}$$

In the same way, the mass concentration $C_i$ of the component $i$ of the particulate matter is as shown in Eq. (23).

$$C_i = \sum_{j=1}^{p} a_{ij} S_j \tag{23}$$

Here, $a_{ij}$ is the density containing component $i$ in the particulate matter which comes from the emission source $j$ to the receptor.

There are two receptor models, the CMB (Chemical Mass Balance) model which handles one piece of observation data and a multivariate model which handles numerous pieces of data. Outlined below is the typical CMB model method which is often used for the analysis of source contribution.

10.6.2 The CMB model

Fig. 10.6.1 shows the concentration range and geometric mean of various elements in the atmosphere gathered from the results of analyzing many particulate matter samples collected in city areas. Table 10.6.1 is an example of measurements of the component concentration of the particles discharged from the principal emission source in a city area. The basic concept of CMB is that the contribution concentration of the emission source is determined by obtaining much of the chemical components concentration from the samples which were gathered at the measurement station. Analysis is made with the observation on the characteristic features of a chemical component concentration pattern (emission source profile) of particles discharged from the emission source as shown in Table 10.6.1.

More than 20 kinds of chemical components are supposed to have been to be correctly grasped from the discharged particles of the principle emission source in the city area. Then, as for the concentration of the chemical component of the particulate matter which is observed in the environment, the concentration of the chemical components of these particles from the emission source superpose into a chemical component concentration pattern with the weight percentage of $S_j$. The weight percentage is $S_j$ which is the contributed concentration of the particles discharged from the emission source. When the measured number $n$ of the chemical components is equal to or
higher than the number of the emission sources, theoretically, by solving the simultaneous equation in Eq. (23), the mass contributed concentration of each emission source can be obtained. The tracer element method, linear programming method, the ordinary linear least-squares method, the effective variance least-squares method and so on, are used as the actual calculation techniques.

Fig. 10.6.1 Concentration of each element in the suspended particulate matter in the atmosphere in a metropolitan area.
Table 10.6.1 Component concentration of discharged particles from the main emission sources in the city area (Unit: μ g/g unless indicated otherwise)

<table>
<thead>
<tr>
<th>Element (%)</th>
<th>Soil</th>
<th>Sea salt</th>
<th>Iron/ steel Ind.</th>
<th>Trash burning</th>
<th>Petroleum burning</th>
<th>Auto exhaust gas leaded gasoline</th>
<th>Diesel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na (9%)</td>
<td>1.3</td>
<td>30.42</td>
<td>1.4</td>
<td>12</td>
<td>1.0</td>
<td>0.027</td>
<td>0.10</td>
</tr>
<tr>
<td>Al (9%)</td>
<td>6.8</td>
<td>0.00003</td>
<td>1.0</td>
<td>0.42</td>
<td>0.21</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>S (9%)</td>
<td>2.6</td>
<td>2.48</td>
<td>13</td>
<td>9.6</td>
<td></td>
<td>0.34</td>
<td>-</td>
</tr>
<tr>
<td>Cl (9%)</td>
<td>0.023</td>
<td>55.05</td>
<td>3.4</td>
<td>27</td>
<td>0.092</td>
<td>0.12</td>
<td>0.090</td>
</tr>
<tr>
<td>K (9%)</td>
<td>1.6</td>
<td>1.1</td>
<td>1.3</td>
<td>20</td>
<td>0.085</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>Ca (9%)</td>
<td>1.6</td>
<td>1.2</td>
<td>4.5</td>
<td>1.1</td>
<td>0.085</td>
<td>0.23</td>
<td>0.36</td>
</tr>
<tr>
<td>Se (9%)</td>
<td>0.001</td>
<td>1.3</td>
<td>0.46</td>
<td>0.09</td>
<td>0.23</td>
<td>0.23</td>
<td>0.70</td>
</tr>
<tr>
<td>Ti (9%)</td>
<td>4200</td>
<td>0.029</td>
<td>1000</td>
<td>900</td>
<td>740</td>
<td>190</td>
<td>-</td>
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<tr>
<td>V (9%)</td>
<td>6.1</td>
<td>0.058</td>
<td>130</td>
<td>27</td>
<td>9200</td>
<td>2.4</td>
<td>9.6</td>
</tr>
<tr>
<td>Cr (9%)</td>
<td>46</td>
<td>0.001</td>
<td>3200</td>
<td>850</td>
<td>210</td>
<td>16</td>
<td>12</td>
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<tr>
<td>Mn (9%)</td>
<td>790</td>
<td>0.058</td>
<td>22000</td>
<td>330</td>
<td>120</td>
<td>36</td>
<td>44</td>
</tr>
<tr>
<td>Fe (9%)</td>
<td>3.9</td>
<td>0.000029</td>
<td>15.7</td>
<td>0.62</td>
<td>0.46</td>
<td>0.41</td>
<td>0.11</td>
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<tr>
<td>Co (9%)</td>
<td>8.9</td>
<td>0.014</td>
<td>44</td>
<td>21</td>
<td>31</td>
<td>1.8</td>
<td>0.70</td>
</tr>
<tr>
<td>Ni (9%)</td>
<td>0.014</td>
<td>2900</td>
<td>-</td>
<td>4900</td>
<td>39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cu (9%)</td>
<td>-</td>
<td>0.017</td>
<td>3700</td>
<td>3600</td>
<td>-</td>
<td>190</td>
<td>-</td>
</tr>
<tr>
<td>Zn (9%)</td>
<td>290</td>
<td>0.029</td>
<td>52000</td>
<td>26000</td>
<td>400</td>
<td>1500</td>
<td>470</td>
</tr>
<tr>
<td>As (9%)</td>
<td>18</td>
<td>0.029</td>
<td>100</td>
<td>150</td>
<td>23</td>
<td>-</td>
<td>2.8</td>
</tr>
<tr>
<td>Se (9%)</td>
<td>5.2</td>
<td>0.12</td>
<td>51</td>
<td>-48</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Br (9%)</td>
<td>1.3</td>
<td>1900</td>
<td>140</td>
<td>830</td>
<td>8.5</td>
<td>6200</td>
<td>38</td>
</tr>
<tr>
<td>Rb (9%)</td>
<td>86</td>
<td>3.5</td>
<td>77</td>
<td>260</td>
<td>-</td>
<td>-</td>
<td>1.9</td>
</tr>
<tr>
<td>Ag (9%)</td>
<td>-</td>
<td>0.087</td>
<td>54</td>
<td>150</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cd (9%)</td>
<td>-</td>
<td>0.002</td>
<td>250</td>
<td>-500</td>
<td>240</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Sn (9%)</td>
<td>-</td>
<td>0.087</td>
<td>2800</td>
<td>3000</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Sb (9%)</td>
<td>37</td>
<td>0.014</td>
<td>90</td>
<td>610</td>
<td>6.9</td>
<td>2.1</td>
<td>4.7</td>
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<tr>
<td>Cs (9%)</td>
<td>3.2</td>
<td>0.029</td>
<td>-</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>0.21</td>
</tr>
<tr>
<td>Ba (9%)</td>
<td>730</td>
<td>0.86</td>
<td>-</td>
<td>390</td>
<td>920</td>
<td>640</td>
<td>20</td>
</tr>
<tr>
<td>La (9%)</td>
<td>21</td>
<td>0.009</td>
<td>9.8</td>
<td>7.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ce (9%)</td>
<td>49</td>
<td>0.012</td>
<td>69</td>
<td>170</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sm (9%)</td>
<td>4.2</td>
<td>-</td>
<td>0.21</td>
<td>0.49</td>
<td>7.8</td>
<td>-</td>
<td>0.13</td>
</tr>
<tr>
<td>W (9%)</td>
<td>2.5</td>
<td>0.003</td>
<td>47</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pb (9%)</td>
<td>-</td>
<td>0.087</td>
<td>1.4</td>
<td>1.7</td>
<td>0.033</td>
<td>3.7</td>
<td>-</td>
</tr>
<tr>
<td>Th (9%)</td>
<td>9.3</td>
<td>0.020</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.11</td>
</tr>
<tr>
<td>V - C (9%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NV - C (9%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SO42- (9%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NO3- (9%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.78</td>
</tr>
</tbody>
</table>

(1) The tracer element method

The tracer element method is the simplest calculation process and it is assumed that, concerning the chemical component concentration, unique components (the index components) exist in each emission source that do not exist in other emission sources. The contributed concentration $S_j$ of the emission source $j$ which has the index component $t$ can be given by Eq. (24) which is a transformation of Eq. (23).

$$S_j = \frac{C_t}{\alpha_{tj}}$$  \hspace{1cm} (24)
This method is effective only when, regarding the index element, (1) the emission source particles $a_{ij}$ in the receptor are well known and do not change between the emission source and receptor, (2) $C_i$ in the environment sample can be measured with correctness and high precision and (3) the concentration of component $i$ in the receptor originates only from the emission source $j$.

Actually, the above conditions are not completely satisfied and limiting a model to one emission source with one index element means that the other important information contained in the aerosol is not used. This is effective to find the most approximate value or the upper limit value of the source contribution.

(2) The linear programming method

The linear-programming method seeks the combination which maximizes the summation of the source contribution under the conditions given by Eq. (25) and Eq. (26):

\[ 0 \leq S_j \leq M \]

\[ C_i + 3 \sigma_{C_i} \geq \sum_{j=1}^{P} a_{ij} S_j \]

Here, $M$ is the concentration of the particulate matter and $\sigma_{C_i}$ is the measurement error of $C_i$. These conditions are necessary conditions that source contribution should meet, but they are not sufficient and the physical meaning of the end result is not definite.

(3) The ordinary linear least squares method

In the least squares method, the most probable value of the source contribution $S_j$ is sought in the case of $n \geq p$ by minimizing $x^2$ which is calculated in Eq. (27).

\[ x^2 = \sum_{i=1}^{n} \left( \frac{C_i - \sum_{j=1}^{P} a_{ij} S_j}{\sigma_{C_i}} \right)^2 \]

Eq. (23) of CMB becomes Eq. (28) when expressed in a procession.

\[ C = AS \]

Here, $C$ is the component concentration observation value vector of the environment particle of $n \times 1$, $A$ is the component concentration procession of the emission source particle $n \times p$ and $S$ is the emission source contribution concentration vector of $p \times 1$. The answer to the least-square method adopted for Eq. (28) is shown in Eq. (29).

\[ S = (A' W A)^{-1} A' W C \]
In this formula, $W$ is a diagonal matrix and its diagonal component is $\sigma^{-2}$. Also, the $t$ and the $-1$ in the power positions show a transpose matrix and an inverse matrix, respectively. There is no error in the component concentration matrix of the source particle. And then, the estimated error of the emission source contribution which is calculated by the least-square method, supposing that there is an error only in the component concentration observation value of the environment particles is given as the diagonal component of the matrix in Eq. (30).

$$ (A'WA)^{-1} $$ ................................................................. (30)

(4) The effective variance least-squares method

Since calculations are accompanied by an error in the emission source profile data as well as an error in the established data of the component concentration value of environmental particles, both of these errors must be considered. Effective variance least-squares method is solved in Eq. (23) by multiplying each chemical by the weight percentage which is proportional to the precision of the emission source profile data and the environment concentration measurement data of each chemical component. That is, repeat computations to seek $S_j$ which minimizes the function $x^2$ in Eq. (31).

$$ x^2 = \sum_{i=1}^{n} \frac{(C_{ij} - \sum_{j=1}^{k} a_{ij} S_j)^2}{\sigma_{ij}^2 + \sum_{j=1}^{k} \sigma_{ij}^2 S_j^2} $$ ................................................................. (31)

Here, $\sigma_{ij}$ is the error which accompanies measurement of $a_{ij}$ and the denominator on the right side of Eq. (31) is called the effective dispersion $V_{eff}$. The answer in Eq. (31) is expressed by the matrix in Eq. (32).

$$ S = (A'VA)^{-1} A'VC $$ ................................................................. (32)

Here, $V$ is the diagonal matrix which has $V_{eff}$ as the diagonal component. Also, the estimated error of the computed $S_j$ is given as the diagonal component of the matrix in Eq. (33).

$$ (A'VA)^{-1} $$ ................................................................. (33)

10.6.3 Index elements

In CMB, the choice of the index component determines the result. Index components are selected after considering (1) that they are specific to the emission source and distinguishable from other emission sources, (2) that they are measured with high precision and (3) the component concentration does not change in the atmosphere. The components that meet condition number (2) do not always have a high concentration because sensitivity and precision depend on the analysis method used. Components which escape from particles in the atmosphere or those that were released in gas and form particles in the atmosphere do not satisfy condition (3). Actually, among the
optional components that satisfy condition (1), the best component which meets conditions (2) and (3) most is chosen.

The components which are often used from the past as the indexes are Al, Sc, Ti, Th (soil burning and coal combustion), Na (sea salt), V, Ni (fuel oil combustion), Fe, Mn, Zn (iron/steel industry), K, Zn (refuse incineration), Pb, Br, C (car exhaust), and so on. These components are used independently or in combination.
(10.1~5) References


2) About the air pollution real-time supervisory control system, industrial pollution 11, (9) 43-52 (1975)


4) Kondo, H.; The Thermally Induced Local Wind and Surface Inversion over the Kanto Plain on Calm Winter Nights, J. of Applied Meteorology, 34, (6) 1439-1448 (1995)


(10.6) References


2) Environmental Agency, Air Quality Bureau, Air Pollution Regulation Division; The Analysis and Estimate of the Suspended Particulate Matter Pollution, pp.303-314 (1987)